**Лабораторна робота №3**

**з чисельних методів**

**Варіант №6**

**ТК-31**

**Луковського Максима Юрійовича**

## Постановка задачі

1. Степеневим методом з точністю 10-4 знайти максимальне та мінімальне за модулем

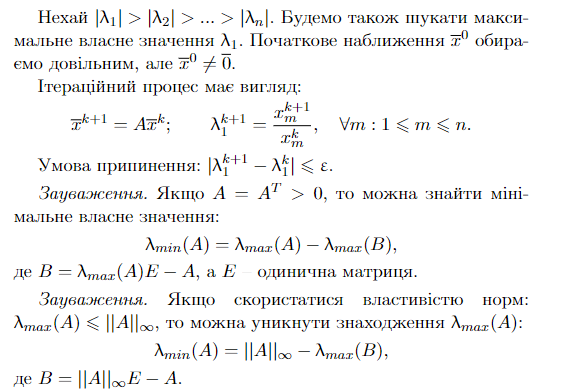
власні значення та відповідні власні вектори заданої матриці.

2. Методом обертання Якобі з точністю 10-4 знайти всі власні значення та відповідні

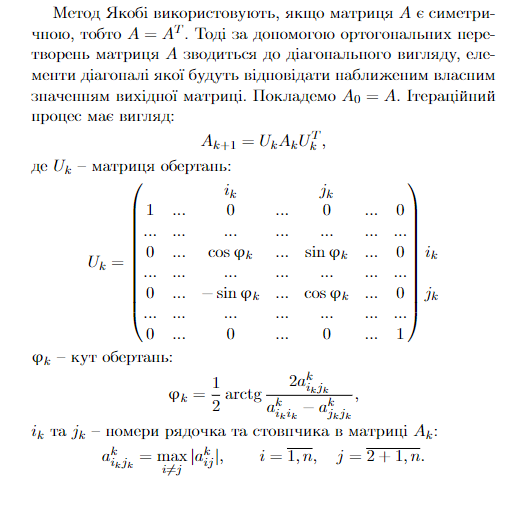
власні вектори заданої симетричної матриці.

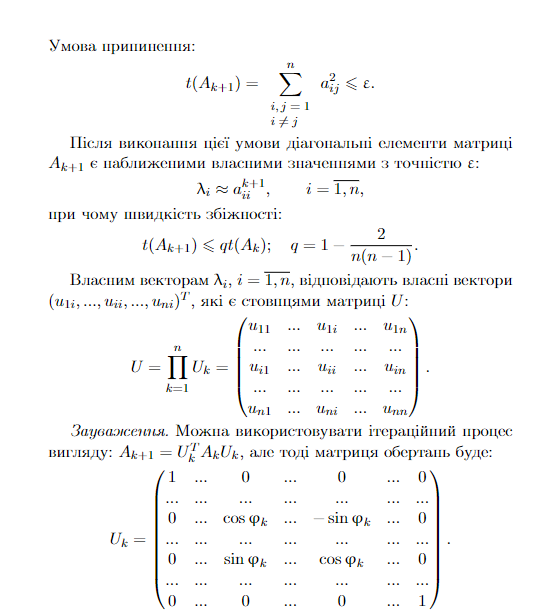
## Теоретична частина

### Степеневий метод



### Метод обертання Якобі:





## Практична частина

### Степеневий метод:

A = AT

A = [5.18, 1.12, 0.95, 1.32, 0.83],

[1.12, 6.28, 2.12, 0.57, 0.91],

[0.95, 2.12, 6.13, 1.29, 1.57],

[1.32, 0.57, 1.29, 5.67, 1.25],

[0.83, 0.91, 1.57, 1.25, 5.21]

Перевірка визначників головних мінорів:

Det(1) = 5.18

Det(2) = 31.28

Det(3) = 167.28

Det(4) = 861.31

Det(5) = 3965.32

Всі головні мінори додатні

B: [[6.880000000000001, -1.12, -0.95, -1.32, -0.83], [-1.12, 5.78, -2.12, -0.57, -0.91], [-0.95, -2.12, 5.930000000000001, -1.29, -1.57], [-1.32, -0.57, -1.29, 6.390000000000001, -1.25], [-0.83, -0.91, -1.57, -1.25, 6.8500000000000005]]

Assessment\_above: 12.06

0

lambda\_k1: 2.6600000000000006

1

lambda\_k1: 4.746503759398497

2

lambda\_k1: 6.463258116381666

3

lambda\_k1: 7.061485242152027

4

lambda\_k1: 7.215698107911887

5

lambda\_k1: 7.261692797896488

6

lambda\_k1: 7.280838594788558

7

lambda\_k1: 7.289593338117425

8

lambda\_k1: 7.290747030865147

9

lambda\_k1: 7.284218033027526

10

lambda\_k1: 7.26914947941944

11

lambda\_k1: 7.244218783692151

12

lambda\_k1: 7.207539959805586

13

lambda\_k1: 7.156394931589496

14

lambda\_k1: 7.086767158997

15

lambda\_k1: 6.99252416290706

16

lambda\_k1: 6.863897808861533

17

lambda\_k1: 6.684434573137759

18

lambda\_k1: 6.424248580612576

19

lambda\_k1: 6.02301667658688

20

lambda\_k1: 5.338338476664424

21

lambda\_k1: 3.934862707525272

22

lambda\_k1: -0.46429014803251434

23

lambda\_k1: 158.01341036851292

24

lambda\_k1: 15.842134488963257

25

lambda\_k1: 12.102899564662437

26

lambda\_k1: 10.818154990960764

27

lambda\_k1: 10.171234183812794

28

lambda\_k1: 9.783260103669836

29

lambda\_k1: 9.525703676036928

30

lambda\_k1: 9.342934526283285

31

lambda\_k1: 9.206963963819817

32

lambda\_k1: 9.10217905350875

33

lambda\_k1: 9.019187074720165

34

lambda\_k1: 8.952001226597636

35

lambda\_k1: 8.896627254805216

36

lambda\_k1: 8.85029977048203

37

lambda\_k1: 8.811044654655158

38

lambda\_k1: 8.777415913062327

39

lambda\_k1: 8.748330909925413

40

lambda\_k1: 8.72296359527344

41

lambda\_k1: 8.700673236702416

42

lambda\_k1: 8.680955609439138

43

lambda\_k1: 8.66340880352175

44

lambda\_k1: 8.647708786771657

45

lambda\_k1: 8.633591625952103

46

lambda\_k1: 8.620840343624062

47

lambda\_k1: 8.609275061008931

48

lambda\_k1: 8.598745508241592

49

lambda\_k1: 8.589125265534385

50

lambda\_k1: 8.580307287031413

51

lambda\_k1: 8.572200386982079

52

lambda\_k1: 8.564726456099377

53

lambda\_k1: 8.557818237775198

54

lambda\_k1: 8.55141753771542

55

lambda\_k1: 8.545473772122167

56

lambda\_k1: 8.539942782519862

57

lambda\_k1: 8.534785862219746

58

lambda\_k1: 8.529968951977265

59

lambda\_k1: 8.525461971821226

60

lambda\_k1: 8.521238263169346

61

lambda\_k1: 8.517274120793267

62

lambda\_k1: 8.513548398389103

63

lambda\_k1: 8.51004217476076

64

lambda\_k1: 8.5067384701618

65

lambda\_k1: 8.503622004336904

66

lambda\_k1: 8.500678989382335

67

lambda\_k1: 8.497896951800442

68

lambda\_k1: 8.495264579128106

69

lambda\_k1: 8.492771587327306

70

lambda\_k1: 8.490408605779454

71

lambda\_k1: 8.488167077256092

72

lambda\_k1: 8.486039170671758

73

lambda\_k1: 8.48401770478

74

lambda\_k1: 8.482096081265786

75

lambda\_k1: 8.480268225929048

76

lambda\_k1: 8.478528536854457

77

lambda\_k1: 8.476871838629197

78

lambda\_k1: 8.475293341809811

79

lambda\_k1: 8.473788606955816

80

lambda\_k1: 8.472353512645949

81

lambda\_k1: 8.470984226975517

82

lambda\_k1: 8.469677182103293

83

lambda\_k1: 8.468429051475672

84

lambda\_k1: 8.467236729406178

85

lambda\_k1: 8.46609731273138

86

lambda\_k1: 8.465008084300955

87

lambda\_k1: 8.463966498091079

88

lambda\_k1: 8.462970165757271

89

lambda\_k1: 8.462016844466062

90

lambda\_k1: 8.461104425864796

91

lambda\_k1: 8.460230926066252

92

lambda\_k1: 8.459394476539618

93

lambda\_k1: 8.458593315812397

94

lambda\_k1: 8.457825781899103

95

lambda\_k1: 8.457090305382351

96

lambda\_k1: 8.45638540308057

97

lambda\_k1: 8.45570967224404

98

lambda\_k1: 8.455061785227418

99

lambda\_k1: 8.454440484592785

100

lambda\_k1: 8.453844578602173

101

lambda\_k1: 8.453272937063

102

lambda\_k1: 8.45272448749375

103

lambda\_k1: 8.452198211580676

104

lambda\_k1: 8.451693141899327

105

lambda\_k1: 8.451208358877404

106

lambda\_k1: 8.450742987977826

107

lambda\_k1: 8.450296197083032

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lambda\_k1: 8.449867194063373

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lambda\_k1: 8.449455224514164

110

lambda\_k1: 8.449059569647453

111

lambda\_k1: 8.448679544325866

112

lambda\_k1: 8.44831449522711

113

lambda\_k1: 8.447963799128754

114

lambda\_k1: 8.447626861303876

115

lambda\_k1: 8.44730311401902

116

lambda\_k1: 8.446992015126652

117

lambda\_k1: 8.446693046745027

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lambda\_k1: 8.446405714018985

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lambda\_k1: 8.446129543955712

120

lambda\_k1: 8.44586408433013

121

lambda\_k1: 8.445608902654875

122

lambda\_k1: 8.445363585210345

123

lambda\_k1: 8.445127736130667

124

lambda\_k1: 8.444900976541716

125

lambda\_k1: 8.444682943747669

126

lambda\_k1: 8.444473290462872

127

lambda\_k1: 8.44427168408597

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lambda\_k1: 8.44407780601363

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lambda\_k1: 8.443891350991185

130

lambda\_k1: 8.443712026497996

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lambda\_k1: 8.4435395521652

132

lambda\_k1: 8.443373659223932

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lambda\_k1: 8.443214089982105

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lambda\_k1: 8.443060597328019

135

lambda\_k1: 8.442912944259184

136

lambda\_k1: 8.442770903434873

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lambda\_k1: 8.442634256750987

138

lambda\_k1: 8.442502794935953

139

lambda\_k1: 8.442376317166449

140

lambda\_k1: 8.442254630701807

141

lambda\_k1: 8.442137550536053

142

lambda\_k1: 8.442024899066622

143

lambda\_k1: 8.441916505778773

144

lambda\_k1: 8.441812206944928

145

lambda\_k1: 8.44171184533803

146

lambda\_k1: 8.44161526995826

Lambda max B: 8.44161526995826

0

lambda\_k1: 9.4

1

lambda\_k1: 9.990436170212766

2

lambda\_k1: 10.314168082027386

3

lambda\_k1: 10.477358786912417

4

lambda\_k1: 10.556621874454935

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lambda\_k1: 10.59467388372911

6

lambda\_k1: 10.612960588127283

7

lambda\_k1: 10.621810271054047

8

lambda\_k1: 10.6261337602597

9

lambda\_k1: 10.62826760468856

10

lambda\_k1: 10.629331303071472

11

lambda\_k1: 10.629866484970195

12

lambda\_k1: 10.630138012916694

13

lambda\_k1: 10.630276793189145

14

lambda\_k1: 10.63034817957611

Lambda max A: 10.63034817957611

Eigenvector: [0.3558285251395442, 0.4985916131455326, 0.5603509668966932, 0.3964749879947061, 0.3919270873395854]

Check for eigenvector lambda max A

A \* v = [3.782594252142562, 5.300271721167229, 5.956781007464145, 4.214665661078351, 4.166340921888687]

lambda \* v = [3.782581114458406, 5.300202447153528, 5.956725880873975, 4.214667166876984, 4.166321399426929]

Lambda min A: 3.6183847300417398

Eigenvector: [-0.43564039079287875, 0.4999591494461038, -0.6023531565757, 0.3692308572384091, 0.24717918120512475]

Check for eigenvector lambda min A

A \* v = [-1.576355023719437, 1.8102321724155805, -2.181990703906942, 1.3354087644032115, 0.8970289514407275]

lambda \* v = [-1.5763145378343686, 1.8090445520004381, -2.179545463845954, 1.336019295691681, 0.8943893748568436]

Отже отримали такі результати:  
Максимальне власне число: 10.63034817957611

Власний вектор: 0.3558285251395442, 0.4985916131455326, 0.5603509668966932, 0.3964749879947061, 0.3919270873395854

Мінімальне власне число: 3.6183847300417398

Власний вектор: -0.43564039079287875, 0.4999591494461038, -0.6023531565757, 0.3692308572384091, 0.24717918120512475

### Метод обертання Якобі:

A = [5.18, 1.12, 0.95, 1.32, 0.83],

[1.12, 6.28, 2.12, 0.57, 0.91],

[0.95, 2.12, 6.13, 1.29, 1.57],

[1.32, 0.57, 1.29, 5.67, 1.25],

[0.83, 0.91, 1.57, 1.25, 5.21]

b = [8.64, 3.21, 1.83, 3.25, 7.4 ]

A = AT отже метод Якобі застосовний

Розпочнемо ітерації:

Iteration 1:

Max off-diagonal element: A[1][2] = 2.12

Rotation angle phi: 0.7677168580819733

Updated matrix A:

['5.18', '1.4656075703891167', '-0.09431038976756811', '1.32', '0.83']

['1.4656075703891167', '8.326326236107967', '8.881784197001252e-16', '1.3060116489059848', '1.74509944104464']

['-0.09431038976756811', '0.0', '4.083673763892032', '0.5322908724765725', '0.49762228734823044']

['1.32', '1.3060116489059848', '0.5322908724765725', '5.67', '1.25']

['0.83', '1.74509944104464', '0.49762228734823044', '1.25', '5.21']

Current eigenvalues : [5.18, 8.326326236107967, 4.083673763892032, 5.67, 5.21]

Iteration 2:

Max off-diagonal element: A[1][4] = 1.74509944104464

Rotation angle phi: 0.42096464372596304

Updated matrix A:

['5.18', '1.6768250405493035', '-0.09431038976756811', '1.32', '0.15862954885114122']

['1.6768250405493035', '9.10765972236826', '0.203348903322017', '1.7027918624074996', '4.440892098500626e-16']

['-0.09431038976756811', '0.2033489033220162', '4.083673763892032', '0.5322908724765725', '0.45417745913180035']

['1.32', '1.7027918624074996', '0.5322908724765725', '5.67', '0.6071789690008442']

['0.15862954885114122', '4.440892098500626e-16', '0.45417745913179997', '0.6071789690008442', '4.428666513739708']

Current eigenvalues : [5.18, 9.10765972236826, 4.083673763892032, 5.67, 4.428666513739708]

Iteration 3:

Max off-diagonal element: A[1][3] = 1.7027918624074996

Rotation angle phi: 0.3903554735180941

Updated matrix A:

['5.18', '2.0529666228286207', '-0.09431038976756811', '0.5826407659654628', '0.15862954885114122']

['2.0529666228286207', '9.808308362510285', '0.39059756649095567', '-4.440892098500626e-16', '0.2310420240443009']

['-0.09431038976756811', '0.3905975664909549', '4.083673763892032', '0.41487093228555005', '0.45417745913180035']

['0.5826407659654628', '2.220446049250313e-16', '0.4148709322855497', '4.969351359857975', '0.5615032355404915']

['0.15862954885114122', '0.2310420240443009', '0.45417745913179997', '0.5615032355404915', '4.428666513739708']

Current eigenvalues : [5.18, 9.808308362510285, 4.083673763892032, 4.969351359857975, 4.428666513739708]

Iteration 4:

Max off-diagonal element: A[0][1] = 2.0529666228286207

Rotation angle phi: -0.36283079807463336

Updated matrix A:

['4.40061550700699', '-2.220446049250313e-16', '-0.2268021220881511', '0.5447084387592958', '0.0663001755110374']

['-4.440892098500626e-16', '10.587692855503294', '0.33169520042213235', '0.20679211520082574', '0.2723013723256083']

['-0.22680212208815081', '0.33169520042213163', '4.083673763892032', '0.41487093228555005', '0.45417745913180035']

['0.5447084387592955', '0.20679211520082635', '0.4148709322855497', '4.969351359857975', '0.5615032355404915']

['0.0663001755110374', '0.2723013723256083', '0.45417745913179997', '0.5615032355404915', '4.428666513739708']

Current eigenvalues : [4.40061550700699, 10.587692855503294, 4.083673763892032, 4.969351359857975, 4.428666513739708]

Iteration 5:

Max off-diagonal element: A[3][4] = 0.5615032355404915

Rotation angle phi: 0.5610444335289689

Updated matrix A:

['4.40061550700699', '-2.220446049250313e-16', '-0.2268021220881511', '0.4964809470837692', '-0.23368711070828735']

['-4.440892098500626e-16', '10.587692855503294', '0.33169520042213235', '0.31997446670938823', '0.12052948574387835']

['-0.22680212208815081', '0.33169520042213163', '4.083673763892032', '0.5929257288417892', '0.16381127835510198']

['0.49648094708376905', '0.3199744667093887', '0.5929257288417887', '5.322203055214543', '0.0']

['-0.23368711070828724', '0.12052948574387803', '0.1638112783551018', '-2.220446049250313e-16', '4.075814818383139']

Current eigenvalues : [4.40061550700699, 10.587692855503294, 4.083673763892032, 5.322203055214543, 4.075814818383139]

Iteration 6:

Max off-diagonal element: A[2][3] = 0.5929257288417892

Rotation angle phi: -0.38183660766525845

Updated matrix A:

['4.40061550700699', '-2.220446049250313e-16', '-0.39546967839076264', '0.37621306047822395', '-0.23368711070828735']

['-4.440892098500626e-16', '10.587692855503294', '0.1885764955322068', '0.42052856105361924', '0.12052948574387835']

['-0.3954696783907623', '0.18857649553220596', '3.845588060344444', '0.0', '0.1520138905719748']

['0.37621306047822395', '0.4205285610536194', '-2.220446049250313e-16', '5.560288758762132', '0.06104024893055708']

['-0.23368711070828724', '0.12052948574387803', '0.15201389057197473', '0.06104024893055681', '4.075814818383139']

Current eigenvalues : [4.40061550700699, 10.587692855503294, 3.845588060344444, 5.560288758762132, 4.075814818383139]

Iteration 7:

Max off-diagonal element: A[1][3] = 0.42052856105361924

Rotation angle phi: 0.08287974653372046

Updated matrix A:

['4.40061550700699', '0.031144758670387303', '-0.39546967839076264', '0.37492168633163264', '-0.23368711070828735']

['0.03114475867038708', '10.622626178982577', '0.1879291952745402', '0.0', '0.12516897156638615']

['-0.3954696783907623', '0.18792919527453938', '3.845588060344444', '-0.01561128536258756', '0.1520138905719748']

['0.3749216863316327', '3.3306690738754696e-16', '-0.015611285362587851', '5.525355435282853', '0.050852703764906804']

['-0.23368711070828724', '0.12516897156638582', '0.15201389057197473', '0.05085270376490656', '4.075814818383139']

Current eigenvalues : [4.40061550700699, 10.622626178982577, 3.845588060344444, 5.525355435282853, 4.075814818383139]

Iteration 8:

Max off-diagonal element: A[0][2] = -0.39546967839076264

Rotation angle phi: -0.47945445020687155

Updated matrix A:

['4.60622716405195', '-0.059057752228084015', '0.0', '0.3398494454314286', '-0.2774614002620359']

['-0.059057752228084584', '10.622626178982577', '0.18110648892554645', '0.0', '0.12516897156638615']

['0.0', '0.18110648892554584', '3.6399764032994844', '0.1590985152425305', '0.027075080842833374']

['0.33984944543142875', '3.3306690738754696e-16', '0.15909851524253027', '5.525355435282853', '0.050852703764906804']

['-0.2774614002620358', '0.12516897156638582', '0.027075080842833374', '0.05085270376490656', '4.075814818383139']

Current eigenvalues : [4.60622716405195, 10.622626178982577, 3.6399764032994844, 5.525355435282853, 4.075814818383139]

Iteration 9:

Max off-diagonal element: A[0][3] = 0.3398494454314286

Rotation angle phi: -0.31837482848547394

Updated matrix A:

['4.494217297193508', '-0.05608982785857019', '-0.04980157019119329', '-2.220446049250313e-16', '-0.2794357755740518']

['-0.056089827858570626', '10.622626178982577', '0.18110648892554645', '-0.018486462858798708', '0.12516897156638615']

['-0.049801570191193356', '0.18110648892554584', '3.6399764032994844', '0.15110308123552393', '0.027075080842833374']

['-2.220446049250313e-16', '-0.018486462858798215', '0.15110308123552374', '5.637365302141296', '-0.03855481091769532']

['-0.2794357755740516', '0.12516897156638582', '0.027075080842833374', '-0.03855481091769551', '4.075814818383139']

Current eigenvalues : [4.494217297193508, 10.622626178982577, 3.6399764032994844, 5.637365302141296, 4.075814818383139]

Iteration 10:

Max off-diagonal element: A[0][4] = -0.2794357755740518

Rotation angle phi: -0.4640780016060755

Updated matrix A:

['4.634085551212394', '-0.1061829115391467', '-0.0566530545442685', '0.017257075878321843', '-2.220446049250313e-16']

['-0.10618291153914725', '10.622626178982577', '0.18110648892554645', '-0.018486462858798708', '0.08682470575395289']

['-0.05665305454426856', '0.18110648892554584', '3.6399764032994844', '0.15110308123552393', '0.001920366360477193']

['0.017257075878321756', '-0.018486462858798215', '0.15110308123552374', '5.637365302141296', '-0.03447704710425595']

['2.220446049250313e-16', '0.08682470575395279', '0.0019203663604772242', '-0.03447704710425612', '3.9359465643642535']

Current eigenvalues : [4.634085551212394, 10.622626178982577, 3.6399764032994844, 5.637365302141296, 3.9359465643642535]

Iteration 11:

Max off-diagonal element: A[1][2] = 0.18110648892554645

Rotation angle phi: 0.025913415935300402

Updated matrix A:

['4.634085551212394', '-0.10761517221813814', '-0.05388278018612796', '0.017257075878321843', '-2.220446049250313e-16']

['-0.10761517221813868', '10.627320317518421', '0.0', '-0.01456509754487314', '0.086845313450795']

['-0.05388278018612801', '-6.245004513516506e-16', '3.6352822647636387', '0.15153134461690562', '-0.00032995128749047694']

['0.017257075878321756', '-0.014565097544872653', '0.15153134461690543', '5.637365302141296', '-0.03447704710425595']

['2.220446049250313e-16', '0.08684531345079491', '-0.00032995128749044355', '-0.03447704710425612', '3.9359465643642535']

Current eigenvalues : [4.634085551212394, 10.627320317518421, 3.6352822647636387, 5.637365302141296, 3.9359465643642535]

Iteration 12:

Max off-diagonal element: A[2][3] = 0.15153134461690562

Rotation angle phi: -0.07511656720336765

Updated matrix A:

['4.634085551212394', '-0.10761517221813814', '-0.0550259084837202', '0.013164728025978758', '-2.220446049250313e-16']

['-0.10761517221813868', '10.627320317518421', '0.0010930515278451132', '-0.014524025091177206', '0.086845313450795']

['-0.05502590848372024', '0.0010930515278444537', '3.623878293294976', '1.1102230246251565e-16', '0.002258341774449371']

['0.013164728025978668', '-0.014524025091176767', '-1.6653345369377348e-16', '5.648769273609959', '-0.03440458599243678']

['2.220446049250313e-16', '0.08684531345079491', '0.002258341774449417', '-0.034404585992436953', '3.9359465643642535']

Current eigenvalues : [4.634085551212394, 10.627320317518421, 3.623878293294976, 5.648769273609959, 3.9359465643642535]

Iteration 13:

Max off-diagonal element: A[0][1] = -0.10761517221813814

Rotation angle phi: 0.017948394960652626

Updated matrix A:

['4.632153824161214', '1.3877787807814457e-17', '-0.05499742809704751', '0.012901938665325886', '0.001558650297948398']

['-5.551115123125783e-16', '10.6292520445696', '0.0020804491849994375', '-0.014757958785797732', '0.08683132543970258']

['-0.05499742809704756', '0.0020804491849987783', '3.623878293294976', '1.1102230246251565e-16', '0.002258341774449371']

['0.012901938665325803', '-0.014757958785797292', '-1.6653345369377348e-16', '5.648769273609959', '-0.03440458599243678']

['0.0015586502979488403', '0.08683132543970248', '0.002258341774449417', '-0.034404585992436953', '3.9359465643642535']

Current eigenvalues : [4.632153824161214, 10.6292520445696, 3.623878293294976, 5.648769273609959, 3.9359465643642535]

Iteration 14:

Max off-diagonal element: A[1][4] = 0.08683132543970258

Rotation angle phi: 0.012969951719088815

Updated matrix A:

['4.632153824161214', '2.0215052338727992e-05', '-0.05499742809704751', '0.012901938665325886', '0.0015585192019842336']

['2.0215052338164786e-05', '10.630378305822056', '0.002109563963816167', '-0.015202930822569348', '2.7755575615628914e-17']

['-0.05499742809704756', '0.002109563963815507', '3.623878293294976', '1.1102230246251565e-16', '0.0022311692594144932']

['0.012901938665325803', '-0.015202930822568904', '-1.6653345369377348e-16', '5.648769273609959', '-0.03421028762285481']

['0.001558519201984683', '-1.0408340855860843e-16', '0.0022311692594145305', '-0.034210287622854975', '3.934820303111797']

Current eigenvalues : [4.632153824161214, 10.630378305822056, 3.623878293294976, 5.648769273609959, 3.934820303111797]

Iteration 15:

Max off-diagonal element: A[0][2] = -0.05499742809704751

Rotation angle phi: -0.054331177334330634

Updated matrix A:

['4.635144842798955', '-9.437349035975516e-05', '5.551115123125783e-17', '0.012882900882208823', '0.001435057066675885']

['-9.437349036035338e-05', '10.630378305822056', '0.002107548910485832', '-0.015202930822569348', '2.7755575615628914e-17']

['-2.7755575615628914e-17', '0.0021075489104852036', '3.620887274657235', '0.0007006327019303772', '0.0023125115313211726']

['0.012882900882208753', '-0.015202930822568904', '0.0007006327019300956', '5.648769273609959', '-0.03421028762285481']

['0.001435057066676332', '-1.0408340855860843e-16', '0.002312511531321234', '-0.034210287622854975', '3.934820303111797']

Current eigenvalues : [4.635144842798955, 10.630378305822056, 3.620887274657235, 5.648769273609959, 3.934820303111797]

Iteration 16:

Max off-diagonal element: A[3][4] = -0.03421028762285481

Rotation angle phi: -0.019949329374295444

Updated matrix A:

['4.635144842798955', '-9.437349035975516e-05', '5.551115123125783e-17', '0.012851710898976026', '0.0016917597035554708']

['-9.437349036035338e-05', '10.630378305822056', '0.002107548910485832', '-0.015199905724056639', '-0.00030326815793830044']

['-2.7755575615628914e-17', '0.0021075489104852036', '3.620887274657235', '0.0006543632948358446', '0.0023260276103002804']

['0.012851710898975948', '-0.015199905724056193', '0.0006543632948355618', '5.649451836455958', '0.0']

['0.0016917597035559164', '-0.0003032681579384234', '0.0023260276103003364', '-1.8041124150158794e-16', '3.9341377402657987']

Current eigenvalues : [4.635144842798955, 10.630378305822056, 3.620887274657235, 5.649451836455958, 3.9341377402657987]

Iteration 17:

Max off-diagonal element: A[1][3] = -0.015199905724056639

Rotation angle phi: -0.0030515842969498917

Updated matrix A:

['4.635144842798955', '-0.00013359106924999275', '5.551115123125783e-17', '0.012851363072170925', '0.0016917597035554708']

['-0.0001335910692505907', '10.630424689759655', '0.0021055422559141694', '0.0', '-0.0003032667458975722']

['-2.7755575615628914e-17', '0.00210554225591354', '3.620887274657235', '0.0006607916012468008', '0.0023260276103002804']

['0.012851363072170845', '4.440892098500626e-16', '0.0006607916012465199', '5.649405452518357', '-9.254469122085538e-07']

['0.0016917597035559164', '-0.0003032667458976946', '0.0023260276103003364', '-9.254469123893393e-07', '3.9341377402657987']

Current eigenvalues : [4.635144842798955, 10.630424689759655, 3.620887274657235, 5.649405452518357, 3.9341377402657987]

Iteration 18:

Max off-diagonal element: A[0][3] = 0.012851363072170925

Rotation angle phi: -0.012667960315159765

Updated matrix A:

['4.634982033532409', '-0.00013358035023173812', '-8.370657893618562e-06', '6.938893903907228e-18', '0.001691635684136148']

['-0.0001335803502323304', '10.630424689759655', '0.0021055422559141694', '-1.6922811007848075e-06', '-0.0003032667458975722']

['-8.370657893705381e-06', '0.00210554225591354', '3.620887274657235', '0.0006607385809567484', '0.0023260276103002804']

['-6.938893903907228e-17', '-1.692281100333179e-06', '0.0006607385809564685', '5.649568261784904', '2.050519893364707e-05']

['0.0016916356841365958', '-0.0003032667458976946', '0.0023260276103003364', '2.0505198933471944e-05', '3.9341377402657987']

Current eigenvalues : [4.634982033532409, 10.630424689759655, 3.620887274657235, 5.649568261784904, 3.9341377402657987]

Iteration 19:

Max off-diagonal element: A[2][4] = 0.0023260276103002804

Rotation angle phi: -0.007424910372986557

Updated matrix A:

['4.634982033532409', '-0.00013358035023173812', '-2.0930555092998477e-05', '6.938893903907228e-18', '0.001691526904196278']

['-0.0001335803502323304', '10.630424689759655', '0.0021077359253698145', '-1.6922811007848075e-06', '-0.00028762506760170257']

['-2.093055509308862e-05', '0.002107735925369186', '3.620870003793325', '0.0006605681201492959', '-3.469446951953614e-18']

['-6.938893903907228e-17', '-1.692281100333179e-06', '0.0006605681201490147', '5.649568261784904', '2.5410513384678714e-05']

['0.0016915269041967251', '-0.00028762506760182964', '5.204170427930421e-17', '2.5410513384505672e-05', '3.934155011129708']

Current eigenvalues : [4.634982033532409, 10.630424689759655, 3.620870003793325, 5.649568261784904, 3.934155011129708]

Iteration 20:

Max off-diagonal element: A[1][2] = 0.0021077359253698145

Rotation angle phi: 0.0003006946611727933

Updated matrix A:

['4.634982033532409', '-0.00013358663789882924', '-2.0890387249209585e-05', '6.938893903907228e-18', '0.001691526904196278']

['-0.00013358663789942154', '10.630425323544614', '4.336808689942018e-19', '-1.4936517202024938e-06', '-0.0002876250545985646']

['-2.0890387249299547e-05', '-6.281867387381013e-16', '3.6208693700083656', '0.0006605685991457947', '8.648732094050155e-08']

['-6.938893903907228e-17', '-1.4936517197509498e-06', '0.0006605685991455133', '5.649568261784904', '2.5410513384678714e-05']

['0.0016915269041967251', '-0.0002876250545986917', '8.64873209960509e-08', '2.5410513384505672e-05', '3.934155011129708']

Current eigenvalues : [4.634982033532409, 10.630425323544614, 3.6208693700083656, 5.649568261784904, 3.934155011129708]

Iteration 21:

Max off-diagonal element: A[0][4] = 0.001691526904196278

Rotation angle phi: 0.002413596667318451

Updated matrix A:

['4.6349861162040344', '-0.00013428045899715546', '-2.0890117655990654e-05', '6.133067088014427e-08', '0.0']

['-0.00013428045899774744', '10.630425323544614', '4.336808689942018e-19', '-1.4936517202024938e-06', '-0.0002873017928754583']

['-2.0890117656080748e-05', '-6.281867387381013e-16', '3.6208693700083656', '0.0006605685991457947', '1.3690798911667668e-07']

['6.133067080423432e-08', '-1.4936517197509498e-06', '0.0006605685991455133', '5.649568261784904', '2.541043937089154e-05']

['4.458239333260394e-16', '-0.00028730179287558675', '1.3690798917200876e-07', '2.5410439370718315e-05', '3.934150928458082']

Current eigenvalues : [4.6349861162040344, 10.630425323544614, 3.6208693700083656, 5.649568261784904, 3.934150928458082]

Iteration 22:

Max off-diagonal element: A[2][3] = 0.0006605685991457947

Rotation angle phi: -0.00032561190250693763

Updated matrix A:

['4.6349861162040344', '-0.00013428045899715546', '-2.089013651856911e-05', '5.452859679554528e-08', '0.0']

['-0.00013428045899774744', '10.630425323544614', '4.863507701375046e-10', '-1.4936516410216932e-06', '-0.0002873017928754583']

['-2.089013651865918e-05', '4.863501413700903e-10', '3.62086915491936', '0.0', '1.2863404049808293e-07']

['5.452859671960599e-08', '-1.493651640570354e-06', '-2.8167572441173405e-16', '5.6495684768739105', '2.5410482602714653e-05']

['4.458239333260394e-16', '-0.00028730179287558675', '1.2863404055347142e-07', '2.5410482602541445e-05', '3.934150928458082']

Current eigenvalues : [4.6349861162040344, 10.630425323544614, 3.62086915491936, 5.6495684768739105, 3.934150928458082]

Iteration 23:

Max off-diagonal element: A[1][4] = -0.0002873017928754583

Rotation angle phi: -4.29047221214695e-05

Updated matrix A:

['4.6349861162040344', '-0.0001342804588735627', '-2.089013651856911e-05', '5.452859679554528e-08', '-5.761265777848763e-09']

['-0.0001342804588741547', '10.630425335871218', '4.808317619262483e-10', '-1.4947418693416247e-06', '5.421010862427522e-20']

['-2.089013651865918e-05', '4.808311331612111e-10', '3.62086915491936', '0.0', '1.2863406124640484e-07']

['5.452859671960599e-08', '-1.4947418688902931e-06', '-2.8167572441173405e-16', '5.6495684768739105', '2.5410418494618085e-05']

['-5.761265332050228e-09', '-1.2847795743953228e-16', '1.286340613018203e-07', '2.5410418494444857e-05', '3.9341509161314776']

Current eigenvalues : [4.6349861162040344, 10.630425335871218, 3.62086915491936, 5.6495684768739105, 3.9341509161314776]

Iteration 24:

Max off-diagonal element: A[0][1] = -0.0001342804588735627

Rotation angle phi: 2.2397101173715953e-05

Updated matrix A:

['4.6349861131965415', '-1.3552527156068805e-20', '-2.089013650256031e-05', '5.449511889699525e-08', '-5.7612657764037505e-09']

['-5.920557013400218e-16', '10.63042533887871', '9.487102629056622e-10', '-1.49474309024922e-06', '1.2903570671446665e-13']

['-2.0890136502650395e-05', '9.487096341426424e-10', '3.62086915491936', '0.0', '1.2863406124640484e-07']

['5.449511882106607e-08', '-1.4947430897978868e-06', '-2.8167572441173405e-16', '5.6495684768739105', '2.5410418494618085e-05']

['-5.761265330608094e-09', '1.2890716456235588e-13', '1.286340613018203e-07', '2.5410418494444857e-05', '3.9341509161314776']

Current eigenvalues : [4.6349861131965415, 10.63042533887871, 3.62086915491936, 5.6495684768739105, 3.9341509161314776]

Eigenvalues and eigenvectors:

Eigenvalue 4.6350 - Eigenvector: [-0.4981859955396406, 0.056728088102237936, -0.411156712223616, 0.31348951195007385, 0.6937341866732625]

Eigenvalue 10.6304 - Eigenvector: [0.3919249839893491, 0.39646873756431206, 0.5603532593709806, 0.49859762514039796, 0.3558257719387154]

Eigenvalue 3.6209 - Eigenvector: [-0.24071359263207034, -0.3741252460574774, 0.5993588416905669, -0.5001610268571262, 0.43897060571883456]

Eigenvalue 5.6496 - Eigenvector: [0.23092429724269714, 0.6500567804904814, -0.22558769784103533, -0.6345162489445793, 0.26570561057192466]

Eigenvalue 3.9342 - Eigenvector: [0.6978083232795765, -0.5263562924058262, -0.3268540227317216, -0.01878326755880228, 0.3589237162552014]

Отже отримали такі результати:

Власні значення та власні вектори:

Власне значення 4.6350 — Власний вектор:

[-0.4982, 0.0567, -0.4112, 0.3135, 0.6937]

Власне значення 10.6304 — Власний вектор:

[0.3919, 0.3965, 0.5604, 0.4986, 0.3558]

Власне значення 3.6209 — Власний вектор:

[-0.2407, -0.3741, 0.5994, -0.5002, 0.4390]

Власне значення 5.6496 — Власний вектор:

[0.2309, 0.6501, -0.2256, -0.6345, 0.2657]

Власне значення 3.9342 — Власний вектор:

[0.6978, -0.5264, -0.3269, -0.0188, 0.3589]

## 

## 

## Висновки

У ході виконання лабораторної роботи були використані степеневий метод та метод обертання Якобі для знаходження власних значень та відповідних власних векторів матриці. Обидва методи з точністю 10⁻⁴ дали схожі результати, що підтверджує правильність проведених обчислень.

Максимальні та мінімальні власні значення, а також відповідні їм вектори, отримані обома методами, виявились близькими за значенням.

## Додатково:

Степеневий метод:

import math

A = [[5.18, 1.12, 0.95, 1.32, 0.83],

[1.12, 6.28, 2.12, 0.57, 0.91],

[0.95, 2.12, 6.13, 1.29, 1.57],

[1.32, 0.57, 1.29, 5.67, 1.25],

[0.83, 0.91, 1.57, 1.25, 5.21]

]

b = [1]\*len(A)

def check\_eigenvector(A, eigenvalue, eigenvector):

Av = matrix\_vector\_multiply(A, eigenvector)

scaled\_vector = [eigenvalue \* x for x in eigenvector]

print("A \* v = ", Av)

print("lambda \* v = ", scaled\_vector)

difference = [Av[i] - scaled\_vector[i] for i in range(len(Av))]

return difference

def dot\_product(v1, v2):

return sum(x \* y for x, y in zip(v1, v2))

def vector\_norm(v):

return math.sqrt(sum(x \* x for x in v))

def matrix\_vector\_multiply(matrix, vector):

return [dot\_product(row, vector) for row in matrix]

def power\_method(A, b, epsilon=1e-4, max\_iterations=1000):

lambda\_k = 0

b\_k = b[:]

for i in range(max\_iterations):

print(i)

b\_k1 = matrix\_vector\_multiply(A, b\_k)

m = 0

lambda\_k1 = b\_k1[m] / b\_k[m]

print('lambda\_k1: ', lambda\_k1)

norm\_b\_k1 = vector\_norm(b\_k1)

b\_k1 = [x / norm\_b\_k1 for x in b\_k1]

if abs(lambda\_k1 - lambda\_k) <= epsilon:

return lambda\_k1, b\_k1

lambda\_k = lambda\_k1

b\_k = b\_k1[:]

return lambda\_k1, b\_k1

def matrix\_infinity\_norm(A):

return max(sum(abs(x) for x in row) for row in A)

def build\_matrix\_B(A, norm\_A\_inf):

size = len(A)

B = [[(norm\_A\_inf if i == j else 0) - A[i][j] for j in range(size)] for i in range(size)]

return B

def check\_determinants(A):

n = len(A)

for i in range(1, n + 1):

submatrix = [row[:i] for row in A[:i]]

det = determinant(submatrix)

print(f"Det({i}) = {det:.2f}")

if det <= 0:

return False

return True

def determinant(A):

n = len(A)

if n == 1:

return A[0][0]

if n == 2:

return A[0][0] \* A[1][1] - A[0][1] \* A[1][0]

det = 0

for j in range(n):

submatrix = [row[:j] + row[j+1:] for row in A[1:]]

sign = (-1) \*\* j

det += sign \* A[0][j] \* determinant(submatrix)

return det

print("\nПеревірка визначників головних мінорів:")

if check\_determinants(A):

print("Всі головні мінори додатні")

else:

print("Не всі головні мінори додатні")

assessment\_above = matrix\_infinity\_norm(A)

B = build\_matrix\_B(A,assessment\_above)

print('B: ', B)

print('Assessment\_above: ',assessment\_above)

lambda\_max\_B, vector\_max\_B = power\_method(B, b)

print('Lambda max B: ', lambda\_max\_B)

lambda\_max\_A, vector\_max\_A = power\_method(A,b)

print('Lambda max A: ', lambda\_max\_A)

print('Eigenvector: ', vector\_max\_A )

print('\n')

print("Check for eigenvector lambda max A")

check\_eigenvector(A, lambda\_max\_A, vector\_max\_A)

print('\n')

lambda\_min\_A = assessment\_above - lambda\_max\_B

print('Lambda min A: ', lambda\_min\_A, )

print('Eigenvector: ', vector\_max\_B )

print('\n')

print("Check for eigenvector lambda min A")

check\_eigenvector(A, lambda\_min\_A, vector\_max\_B)

Метод Якобі:

import math

def identity\_matrix(n):

return [[1 if i == j else 0 for j in range(n)] for i in range(n)]

def matrix\_multiply(A, B):

n = len(A)

result = [[0] \* n for \_ in range(n)]

for i in range(n):

for j in range(n):

for k in range(n):

result[i][j] += A[i][k] \* B[k][j]

return result

def transpose\_matrix(A):

return [[A[j][i] for j in range(len(A))] for i in range(len(A))]

def matrix\_vector\_multiply(matrix, vector):

return [sum(matrix[i][j] \* vector[j] for j in range(len(vector))) for i in range(len(matrix))]

def max\_off\_diagonal(A):

n = len(A)

max\_val = 0

p, q = 0, 0

for i in range(n):

for j in range(i+1, n):

if abs(A[i][j]) > abs(max\_val):

max\_val = A[i][j]

p, q = i, j

return p, q, max\_val

def jacobi\_method(A, epsilon=1e-4):

n = len(A)

U = identity\_matrix(n)

iterations = 0

while True:

p, q, max\_val = max\_off\_diagonal(A)

if abs(max\_val) < epsilon:

break

if A[p][p] == A[q][q]:

phi = math.pi / 4

else:

phi = 0.5 \* math.atan(2 \* A[p][q] / (A[p][p] - A[q][q]))

U\_pq = identity\_matrix(n)

U\_pq[p][p] = math.cos(phi)

U\_pq[q][q] = math.cos(phi)

U\_pq[p][q] = -math.sin(phi)

U\_pq[q][p] = math.sin(phi)

U\_pq\_T = transpose\_matrix(U\_pq)

A = matrix\_multiply(matrix\_multiply(U\_pq\_T, A), U\_pq)

U = matrix\_multiply(U, U\_pq)

iterations += 1

# Виведення на кожній ітерації

print(f"\nIteration {iterations}:")

print(f"Max off-diagonal element: A[{p}][{q}] = {max\_val}")

print(f"Rotation angle phi: {phi}")

print("Updated matrix A:")

for row in A:

print(["{}".format(x) for x in row])

print("Current eigenvalues :", [A[i][i] for i in range(n)])

if iterations > 10000:

break

eigenvalues = [A[i][i] for i in range(n)]

eigenvectors = [[U[i][j] for i in range(n)] for j in range(n)]

return eigenvalues, eigenvectors

A = [[5.18, 1.12, 0.95, 1.32, 0.83],

[1.12, 6.28, 2.12, 0.57, 0.91],

[0.95, 2.12, 6.13, 1.29, 1.57],

[1.32, 0.57, 1.29, 5.67, 1.25],

[0.83, 0.91, 1.57, 1.25, 5.21]

]

eigenvalues, eigenvectors = jacobi\_method(A)

print("\nEigenvalues and eigenvectors:")

for i in range(len(eigenvalues)):

transposed\_vector = eigenvectors[i][::-1]

print(f"Eigenvalue {eigenvalues[i]:.4f} - Eigenvector: {transposed\_vector}")